A CALCULATION OF THE TEMPERATURE OF CONTACT SURFACES IN A HIGH-POWER DISCHARGE OF ELECTRIC CURRENT OF COMMERCIAL FREQUENCY

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Thermal processes occuring at the surface of disconnected electric contacts subjected to a high-power electric arc are considered.

In [1], it was proposed that a calculation of the thermal behavior of an electrode which is in the form of a bar can proceed with the following assumptions: the energy input from the arc is uniformly distributed over the ends of the contact surfaces; the temperature is constant in each cross section, i.e., the one-dimensional thermal conductivity equation is a sufficient approximation to the problem. The results of calculations based on these assumptions for a dc arc were given in [1]. In this paper, we explore the possibility of using this method for an arc produced by a current alternating at commerical frequency.

Let us assume that the energy flowing to the contacts is evenly divided between the anode and cathode. This assumption is based on the fact that there is little difference in the erosion of the cathode and anode [2, 3]. The density of thermal current incident on the surface of the contact will, therefore, be [4]:

$$q = \frac{U_\Sigma I_M}{S} |\sin \omega t|. \tag{1}$$

The cooling of the bars by radiation and thermal exchange with the side surfaces can be neglected to a first approximation [1, 5]. The initial temperature of the bar is taken to be zero. To solve the basic thermal conduction equation

$$\frac{\partial^2 \vartheta}{\partial x^2} - \frac{1}{a} \frac{\partial \vartheta}{\partial t} = 0$$

given the indicated boundary and initial conditions $|\sin \omega t|$, we expand in a Fourier series and retain the first two terms of the series:

$$q = \frac{2U_\Sigma I_M}{\pi S} \left(1 - \frac{2}{3} \cos 2\omega t \right). \tag{2}$$

Using the solution of the basic thermal conduction equation for dc and ac flow to the end surfaces of the bar [6], we find that the surface temperature is given by

$$\vartheta = \frac{4U_\Sigma I_M}{\pi S} \left[\frac{\sqrt{at}}{\lambda \sqrt{\pi}} - \frac{\sqrt{a}}{3\lambda \sqrt{2\omega}} \sin \left(2\omega t + \frac{\pi}{4} \right) + \frac{2a}{3\lambda\pi} \int\limits_0^\infty \frac{au^2}{4\omega^2 + a^2u^4} \exp(-au^2 t)\, du \right]. \tag{3}$$

We introduce the notation

$$\vartheta^* = \frac{\vartheta}{U_\Sigma I_M / S}.$$

From curves of $\vartheta^* = f(t)$ calculated from Eq. (3) for copper and tungsten (Fig. 1), it is possible to determine the surface temperature at any instant if the values of U_Σ, I_M, and S are known. Analysis of these curves shows that if one starts with the assumption of uniform distribution of thermal flux over the

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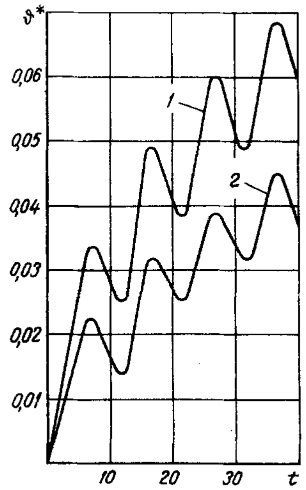

Fig. 1

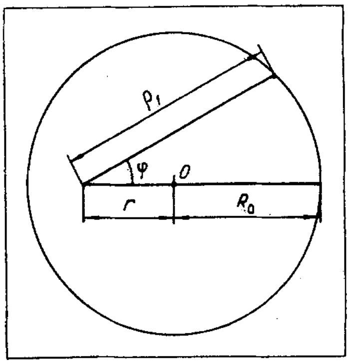

Fig. 2

Fig. 1. Theoretical felations $\vartheta^* = f(t)$ (ϑ^*, deg·cm^2/joule; t, msec). Mean values of thermal coefficients are used: 1) tungsten (a = 0.618 cm^2/sec, λ = 1.67 W/cm·deg); 2) copper (a = 0.832 cm^2/sec, λ = 3.18 W/cm·deg).

Fig. 2. Supporting arc spot.

surface of the bar, it is impossible to explain the experimental data concerning the erosion of the contacts. For example, given a bar diameter 2 cm; a 10 kA current (effective value) and U_Σ = 15 V, melting point temperature is attained on the surface of a tungsten electrode within 24 msec (ϑ^* = 0.05); if the electrode were made of copper its melting point would be reached in 14 msec with a 7kA current. The extent of erosion should be strongly affected by changes in contact dimensions. However, the contact surface is melted as a result of heating by the discharge during a single half-period of the current, and the area affects the results only when it is changed substantially, for example by a transition to a new type of contact [3, 7].

Rapid heating of the contact surfaces to high temperatures can be understood if it is assumed that the energy passes through an area occupying a small portion of the end faces of the contacts. For further analysis, we introduce a number of simplifications:

1) thermal energy passes through a circle of radius R_0;

2) inside this circle, the flux is distributed uniformly and there is no thermal flux outside the circle;

3) after reaching a temperature ϑ_k, the supporting arc spot shifts to a different point;

4) since the time τ during which the supporting arc spot does not move is substantially smaller than a single half-cycle of the current, the flux will be assumed constant during this period τ. The temperature at the supporting arc surface (Fig. 2) can be determined as a function of this and distance from the center of the spot r by the source method [6]

$$\vartheta(r, t) = \int_S \theta(t, \rho)\, dS = \int_0^{2\pi}\int_0^{\rho_1(\varphi)} \frac{2q}{4\pi\lambda\rho}\left[1 - \Phi\left(\frac{\rho}{2\sqrt{at}}\right)\rho d\rho d\varphi\right]. \tag{4}$$

Here $\vartheta(t, \rho)$, the temperature at a point on the surface resulting from the action of a constant point thermal source of intensity qdS located at a distance ρ from the point on the surface and acting for a time t, is given by

$$\theta(t, \rho) = \frac{2q}{4\pi\lambda\rho}\left[1 - \Phi\left(\frac{\rho}{2\sqrt{at}}\right)\right], \tag{5}$$

$\rho_1(\varphi)$ is the distance between the supporting arc spot (which is displaced from the center by a distance r) and the boundary of the spot

$$\rho_1(\varphi) = r\cos\varphi + \sqrt{r^2\cos^2\varphi + R_0^2 - r^2}\,. \tag{6}$$

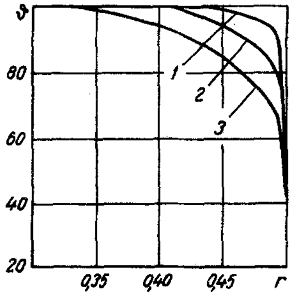

Fig. 3. Theoretical relations for temperature variation in percent on the surface of the supporting arc spot (R_0 = 0.5 cm; r in cm; temperature of the center of the spot is taken to be 100%): 1) t = 1; 2) 2; 3) 10 msec.

From (4) we obtain

$$\vartheta(r,\ t) = \frac{q\sqrt{at}}{\lambda\pi}\int_0^{2\pi} d\varphi \int_0^{\rho_1(\varphi)} \left[1 - \Phi\left(\frac{\rho}{2\sqrt{at}}\right)\right]\frac{d\rho}{2\sqrt{at}}$$

$$= \frac{q\sqrt{at}}{\lambda\pi}\left\{2\sqrt{\pi} + \int_0^{2\pi}\left[\frac{\rho_1(\varphi)}{2\sqrt{at}} - \frac{\rho_1(\varphi)}{2\sqrt{at}}\Phi\left(\frac{\rho_1(\varphi)}{2\sqrt{at}}\right) - \frac{1}{\sqrt{\pi}}\exp\left(-\frac{\rho_1^2(\varphi)}{4at}\right)\right]d\varphi\right\}. \tag{7}$$

We consider certain particular cases.

1) At the center of the spot r = 0, $\rho_1(\varphi) = R_0$

$$\vartheta = \frac{2q\sqrt{at}}{\lambda\sqrt{\pi}}\left\{1 + \sqrt{\pi}\frac{R_0^2}{2\sqrt{at}}\left[1 - \Phi\left(\frac{R_0}{2\sqrt{at}}\right)\right] - \exp\left(-\frac{R_0^2}{4at}\right)\right\}. \tag{8}$$

For t < 10 msec the expression in the braces can be set equal to unity; the error would not exceed a fraction of a percent. Then

$$\vartheta = \frac{2q}{\lambda}\sqrt{\frac{at}{\pi}}. \tag{9}$$

Equation (9) agrees with the expression for the temperature of the surface as calculated on the assumption of one-dimensional linear flow.

2) At the boundary of the circle $r = R_0$

$$\rho_1(\varphi) = R_0\cos\varphi + |R_0\cos\varphi|,$$

$$\rho_1(\varphi) = 2R_0\cos\varphi \quad \text{at} \quad -\frac{\pi}{2} < \varphi < \frac{\pi}{2},$$

$$\rho_1(\varphi) = 0 \quad \text{at} \quad \frac{\pi}{2} < \varphi < \frac{3}{2}\pi,$$

$$\vartheta = \frac{q\sqrt{at}}{\lambda\pi}\left\{\sqrt{\pi} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{2R_0\cos\varphi}{2\sqrt{at}} - \frac{2R_0\cos\varphi}{2\sqrt{at}}\Phi\left(\frac{2R_0\cos\varphi}{2\sqrt{at}}\right) - \frac{1}{\sqrt{\pi}}\exp\left(-\frac{4R_0^2\cos^2\varphi}{4at}\right)\right]d\varphi\right\}. \tag{10}$$

The value of the integral in the expression (10) is always negative. From (9) and (10), it is clear that the temperature on the boundary of the arc spot is lower than the temperature at the center by a factor greater than 2.

The calculations based on (7) showed that this lowering of temperature occurs during the small periods of time when the arc is at the boundary of the spot (Fig. 3) but for an overwhelming part of the time the temperature does not differ significantly from the temperature at the center. By using these expressions, we can determine the maximum spot dimensions which will ensure heating of the contact surface S_g, for a time t_g. For this calculation, we use the mean value of thermal current incident on the contact surface. A certain mean value of τ, determined by Eq. (9), corresponds to this thermal current:

$$\tau = \frac{\pi^3\lambda^2\vartheta_k^2 R_0^4}{4Q^2 a}. \tag{11}$$

Further, we assume that the spot travels once over the whole heated surface and that the surface temperature at the moment when the spot hops onto it is zero, since there is a rapid drop-off of temperature at the edge of the spot (Fig. 3) and that those parts of the surface on which the spot has not yet travelled are hardly heated at all.

Then

$$\tau = \frac{t_g \pi R_0^2}{S_g}. \tag{12}$$

From (11) and (12), we determine the radius of the spot

$$R_0 = \frac{2Q}{\pi \vartheta_k \lambda} \sqrt{\frac{t_g a}{S_g}}. \tag{13}$$

Consider a concrete example. According to experimental data, at a contact consisting of a copper –tungsten of metalloceramic composition, after 10 msec of heating by an arc current of 10 kA, the surface area subjected to erosion S_g is about 3 cm^2. Assume that Q = 150 kW and that the surface temperature is 8300°C, as indicated in [8] for tungsten. Then $\tau = 0.71$ msec and $R_0 = 0.26$ cm. If it is assumed that the circle through which the arc energy is flowing determines the current density at the electrode, then the latter is found to be equal to $4.7 \cdot 10^4$ A/cm^2. Thus, for sufficiently small spot dimensions, it is possible to explain the heating of the contact surface during one half-period of the current, not only up to the melting point, but even to substantially higher temperatures. Of course, these relations allow an evaluation only of the order of magnitude of these parameters because of the simplifications used in deriving these quantities.

In [9], the thermal flux conducted into the interior of the contact from the arc spot was calculated from the equation for the steady state situation

$$Q = 4\lambda\vartheta R_0. \tag{14}$$

The analysis presented above shows that for processes occurring in a short time under the conditions outlined above, the temperature distribution is approximately uniform and the use of Eq. (14) is not justified.

NOTATION

q	is the density of heat flux;
U_Σ	is a certain total equivalent voltage drop determining the value of energy supplied to contacts;
I_M	is the amplitude value of current;
ω	is the circular frequency of current;
S	is the area;
t	is the time;
ϑ	is the temperature;
a	is the thermal diffusivity;
λ	is the thermal conductivity;
$\Phi(x)$	is the probability integral;
$\Phi(x) = 2/\sqrt{\pi}\int_0^x e^{-\alpha^2} d\alpha$;	
Q	is the heat flux;
ρ	is the distance to point heat source;
r	is the distance from point to centre of supporting arc spot;
R_0	is the radius of supporting spot of arc.

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